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CRITERIA FOR THE SEPARATION OF UNSTEADY IDEAL FLUID FLOW PAST A SMOOTH AIRFOIL

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Criteria for the separation of unsteady flow past a closed smooth airfoil are studied using the ideal fluid model and the Brillouin–Villat criterion. The necessary separation conditions are formulated.

Key words: *ideal fluid, separated flow.*

The problem of separated flows past bodies has been one of the main problems in hydrodynamics for more than hundred years. Interest in this problem is motivated not only by its importance (almost all flows past bodies are of separation nature) but also by the extreme complexity and great variety of flow separation phenomena. Despite extensive research, the problem of separated flows is still far from being solved. The most general model of separated flows must take into account fluid viscosity and compressibility and the unsteady nature of the flow. The use of such a model is limited by its complexity and serious computational difficulties. Therefore, use has been made of simpler models.

The model of two-dimensional separated incompressible ideal fluid flow past an open cavity was first considered by Helmholtz (1868) and Kirchhoff (1869). Subsequently, this flow model was complicated and the class of airfoil contours [1–3] was extended. Flow separation was modeled by a line of tangential velocity discontinuity. The point of contact of this line with the airfoil contour defines the flow separation point, which is considered specified. It has been shown that at the separation point, the tangential discontinuity line should have finite curvature (the Brillouin–Villat criterion [1]).

The model of unsteady separated viscous flow past bodies (see, for example, [3–6]) has been studied most extensively. In this model, separated flow arises as a result of boundary-layer separation. The separation condition is the vanishing of the viscous drag force on the body surface, which under a positive pressure gradient leads to return motion of the fluid (the Prandtl criterion).

Real separated flows have a distinct unsteady nature. Therefore, they should be studied using unsteady flow models. Experimental and theoretical studies [6] have shown that in unsteady viscous flows, flow separation occurs not on the body surface but within the boundary layer at the point where the drag force and the longitudinal fluid velocity vanish simultaneously (the Moore–Rott–Sears criterion). Limited theoretical results have been obtained for the model of unsteady separated viscous flows, primarily using methods of asymptotic theories [6, 7]. The model of viscous–inviscid separated flows proposed by Sears turned out to be more promising [8]. In this model, fluid viscosity is taken into account only in the unsteady boundary layer, and outside it and outside the vortex wakes trailing from the body, the flow is considered potential. Vortex wakes are replenished by the wakes formed in the boundary layer, and the flow separation point is defined by the Moore–Rott–Sears criterion [9].

Difficulties arise in modeling unsteady separated flows of an ideal fluid. In this case, the position of the separation point is uncertain. The absence of fluid viscosity prevents the use of the Moore–Rott–Sears criterion, while other conditions for determining the separation point of an ideal fluid flow are not yet available. S. N. Postolovskii approached the solution of this problem. In his dissertation devoted to two-dimensional unsteady separated inviscid incompressible flows (1970), he chooses the initial position of the flow separation point using the minimum pressure

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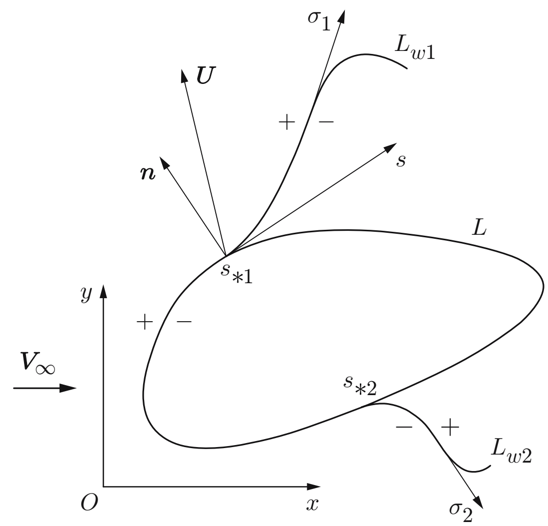

Fig. 1. Separation of two vortex wakes from a smooth airfoil.

condition. At the separation point, the Brillouin–Villat criterion is assumed to be satisfied, the velocity of the separation point is set equal to the fluid velocity at the salient point between the vortex sheet and the airfoil, and continuity of both the pressure and the tangential component of the pressure gradient is required for transition through the vortex sheet. Calculations of separated flow past a circular cylinder using this model are in good agreement with theoretical and experimental data of other authors [10].

In the present paper, we develop Postolovskii's idea of a relationship between the flow separation point with the pressure minimum on the airfoil in the flow. The necessary conditions for the separation of an unsteady ideal fluid flow past an airfoil are formulated and used to derive a nonlinear differential equation for the velocity of motion of the separation point along the airfoil.

1. We consider ideal incompressible flow past a smooth closed airfoil L in Cartesian coordinates Oxy. It is assumed that before the time $t = 0$, the flow past the airfoil L is steady (with a velocity $\boldsymbol{v}_\infty$ at infinite distance from the airfoil) and continuous and that at $t \geq 0$, the fluid motion around L becomes unsteady and separated. At $t > 0$, the points of the airfoil can move at a velocity $\boldsymbol{U}(s,t)$, where s is the arc coordinate reckoned from a certain specified point clockwise along the airfoil. The velocity $\boldsymbol{U}$ should satisfy the condition of zero integral of $\boldsymbol{U}(s,t)$ over the closed airfoil contour L. Physically, this condition implies that for the motion of the points of the contour, the contour length and the volume inside it are conserved. Flow separation occurs in the form of one or several vortex wakes trailing from the airfoil, which are modeled by lines $L_{wk}(t)$ $(k = 1, 2, \ldots)$ of tangential velocity discontinuity. The point of contact of the vortex wake L_{wk} with the contour L defines the flow separation point, which eventually can move along the contour. The arc coordinate of this point will be denoted by $s_{*k}(t)$ $(k = 1, 2, \ldots)$ (see Fig. 1).

We note that the vortex wake line L_{wk} is a cut whose edges L_{wk}^+ and L_{wk}^- are the boundaries of the fluid flow region. For the separated flow model with two vortex wakes L_{w1} and L_{w2} trailing from the airfoil presented in Fig. 1, the flow separation points have the arc coordinates $s = s_{*1} - 0$ and $s = s_{*2} + 0$. On the vortex wake L_{w1}, the streamline of the relative fluid motion that passes through the flow separation point $s_{*1} - 0 \in L$ continues along the edge L_{w1}^+ of the cut L_{w1}. At the other edge L_{w1}^- of the cut, the streamline of the relative motion arrives from the contour L at the point $s = s_{*1} + 0$. A similar flow pattern is observed in the vicinity of the separation point of the vortex wake L_{w2}.

In passing through the vortex wakes L_{wk}, the hydrodynamic pressure and the normal fluid velocity do not vary whereas the tangential velocity component undergoes a discontinuity, whose size γ_{wk} is not known beforehand. The shapes of the vortex wakes L_{wk}, including the position of the separation points, are also unknown. All these unknown functions influence each other in a nonlinear manner and can be determined only by solving the corresponding initial-boundary problem related to the separated unsteady flow past the airfoil.

2. In considering the model of unsteady separated flow of an ideal fluid past a smooth airfoil, one needs to know both the flow separation points (the point of detachment of the vortex wakes) and the velocities of their motion along the airfoil contour. To determine them, it is necessary to formulate the corresponding flow separation conditions.

In real flows, flow separation occurs as a result of interaction of the viscous drag forces, the fluid inertia forces, and the hydrodynamic pressure gradient. In this case, in unsteady flows, as is noted in [6], the separation occurs not from the body surface but outside it in the flow region where the viscous drag forces become far smaller. This allows one to hope for a successful application of the ideal fluid model in studies of unsteady separated flows.

Let us consider physical factors that influence the separation of unsteady ideal fluid flow. The main factor responsible for the separation is probably the pressure gradient, or, to be exact, its tangential component $\partial p/\partial s$, whose magnitude determines the level of the fluid inertia forces along the contour L. At the point of the contour where $\partial p/\partial s = 0$, the tangential component of the fluid inertia force also vanishes. This gives rise to a state of undetermined equilibrium, and only a certain additional impulse is needed to induce flow separation. Such an impulse can be a pressure rise behind the examined point, which changes the sign of the inertia forces and can cause return fluid flow.

In view of the above remarks, we formulate conditions that should be satisfied in the vicinity of the separation point of unsteady flow past a smooth airfoil.

1. At the flow separation point, the tangential component of the pressure gradient vanishes. In the case of two separation points (see Fig. 1), this condition becomes

$$\lim_{s \to s_{*1}-0} \frac{\partial p}{\partial s} = 0, \qquad \lim_{s \to s_{*2}+0} \frac{\partial p}{\partial s} = 0. \tag{1}$$

2. At the flow separation point, the vortex wake L_{wk} has a common tangent with the contour L (the Brillouin–Villat criterion). If this condition for L_{w1} is satisfied, the streamlines for the relative motion of the fluid have a finite curvature at the separation point $s_{*1} - 0$ and an infinitely large curvature at the point $s_{*1} + 0$ at which the streamline of the relative motion forms a return point. According to this, at the point $s_{*1} + 0$ the relative fluid velocity vanishes. For the vortex wake L_{w2}, the pattern of the relative motion becomes opposite: at the point $s_{*2} + 0$, the streamline has a finite curvature, and at the point $s_{*2} - 0$, the velocity is equal to zero.

In the case of violation of the Brillouin–Villat criterion, a vortex wake does not arise since the line L_{wk} and the contour L form salient points at which the relative fluid velocity is equal to zero and a velocity discontinuity is absent.

3. The flow line separation is directed toward increasing pressure along the contour. This condition is a constituent part of the Prandtl criterion for the separation of viscous fluid flow.

We note that the direction of separation of the vortex wakes in Fig. 1 is chosen with allowance for condition 3. It is assumed that the fluid velocity on the contour L increases ahead of the separation points and decreases behind them.

Let us now derive relations that define the position of the separation points of ideal fluid flow using the proposed conditions. We first derive formulas for calculating the pressure at the points of the moving contour and consider the issues of modeling the contour L by a vortex sheet.

3. We assume that the fluid motion outside the contour and the vortex wakes is potential. Then, the fluid velocity $\boldsymbol{v} = \nabla\varphi$, where $\varphi(x, y, t)$ is the velocity potential, and the dynamic pressure is determined by the Cauchy–Lagrange integral (ignoring body forces):

$$p - p_\infty = -\rho\Big[\frac{\partial \varphi}{\partial t} + \frac{1}{2}\,(v^2 - v_\infty^2)\Big]. \tag{2}$$

Here ρ is the fluid density, p_∞ and v_∞ are the pressure and velocity at infinite distance from the contour, $v^2 = v_x^2 + v_y^2 = v_s^2 + v_n^2$, and $\partial/\partial t$ is the operator of differentiation over t at the fixed point (x, y).

We use integral (2) to calculate the hydrodynamic pressure at a point that moves at a velocity $\boldsymbol{v}_e$ in the Cartesian coordinates considered. The operator of differentiation with respect to t at the moving point will be denoted by $\delta/\delta t$. By the definition, we have

$$\frac{\partial}{\partial t} = \frac{\delta}{\delta t} - \boldsymbol{v}_e \cdot \nabla, \qquad \frac{\partial \varphi}{\partial t} = \frac{\delta \varphi}{\delta t} - \boldsymbol{v}_e \cdot \nabla\varphi. \tag{3}$$

Substituting (3) into (2), we obtain

$$p - p_\infty = -\rho\Big[\frac{\delta\varphi}{\delta t} + \frac{1}{2}\,(v^2 - v_\infty^2) - \boldsymbol{v}_e\cdot\boldsymbol{v}\Big]. \tag{4}$$

Let us bring Eq. (4) to a more convenient form. To denote the tangential and normal velocity components, we introduce the subscripts s and n, respectively. At each point of the contour L, the fluid nonpenetration condition $(\boldsymbol{v} - \boldsymbol{U})\cdot\boldsymbol{n} = 0$ should be satisfied [$\boldsymbol{n}$ is the outward normal to L (see Fig. 1)]. Taking into account this condition, we have

$$v^2/2 - \boldsymbol{v}_e\cdot\boldsymbol{v} = (v_s^2 + v_n^2)/2 - (v_{es}v_s + v_{en}v_n) = [(v_s - v_{es})^2 - v_e^2]/2,$$
$$v_e^2 = v_{es}^2 + v_{en}^2, \qquad v_{en} = U_n.$$

As a result, at the point moving at velocity $\boldsymbol{v}_e$ in the Cartesian coordinates, the Cauchy–Lagrange integral (4) becomes

$$p - p_\infty = -\rho\Big\{\frac{\delta\varphi}{\delta t} + \frac{1}{2}\Big[(v_s - v_{es})^2 - v_e^2 - v_\infty^2\Big]\Big\}. \tag{5}$$

4. We model the contour L and the vortex wakes L_{wk} by vortex sheets. In such modeling, it is assumed that inside the closed airfoil contour L there is a fluid which moves together with the contour (at the points of the contour, the velocity of this fluid is $\boldsymbol{U}$). We denote by $v_s^+(s,t)$ and $v_s^-(s,t)$ the limiting values of the fluid velocity with approach to the contour L from the external (+) and internal (−) regions. The vortex sheet $\gamma(s,t)$ that models the contour L has an intensity equal to the discontinuity of the tangential fluid velocity: $\gamma(s,t) = v_s^-(s,t) - v_s^+(s,t)$. Because $v_s^- = U_s$, it follows that

$$\gamma(s,t) = U_s(s,t) - v_s^+(s,t). \tag{6}$$

We note that the intensity of the vortex sheet is directed along the z axis with a positive direction of rotation counterclockwise from the x axis to the y axis.

We use relation (6) to determine $\gamma(s,t)$ in the vicinity of the flow separation points $s_{*1} - 0$ and $s_{*2} + 0$. At the flow separation points, we have

$$\begin{aligned} &v_s^+(s_{*1} - 0, t) = v_s(s_{*1} - 0, t), \qquad \gamma(s_{*1} - 0, t) = U_s(s_{*1}, t) - v_s(s_{*1} - 0, t),\\ &v_s^+(s_{*2} + 0, t) = v_s(s_{*2} + 0, t), \qquad \gamma(s_{*2} + 0, t) = U_s(s_{*2}, t) - v_s(s_{*2} + 0, t). \end{aligned} \tag{7}$$

At the return points $s_{*1} + 0$ and $s_{*2} - 0$ produced by the junction of the edges L_{w1}^- and L_{w2}^+ of the cuts L_{w1} and L_{w2} and the contour L, the relative fluid velocity is equal to zero. Therefore,

$$\begin{aligned} &v_s^+(s_{*1} + 0, t) = v_{es}(s_{*1}, t) = U_s(s_{*1}, t) + \dot{s}_{*1}(t), \qquad \gamma(s_{*1} + 0, t) = -\dot{s}_{*1}(t),\\ &v_s^+(s_{*2} - 0, t) = v_{es}(s_{*2}, t) = U_s(s_{*2}, t) + \dot{s}_{*2}(t), \qquad \gamma(s_{*2} - 0, t) = -\dot{s}_{*2}(t). \end{aligned} \tag{8}$$

The vortex wakes L_{wk} are modeled similarly. We denote the limiting fluid velocity on L_{wk} by $v_{wk\sigma}^+(\sigma,t)$ and $v_{wk\sigma}^-(\sigma,t)$, reckoning the arc coordinates σ along L_{wk} from the point of separation of the vortex wake from the contour L. The intensity of the vortex wake $\gamma_{wk}(\sigma,t)$ is defined by

$$\gamma_{wk}(\sigma,t) = v_{wk\sigma}^-(\sigma,t) - v_{wk\sigma}^+(\sigma,t). \tag{9}$$

According to the Brillouin–Villat criterion, L_{wk} and L have a common tangent. Therefore, for the examined two vortex wakes, we obtain

$$\begin{aligned} &\lim_{\sigma\to 0} v_{w1\sigma}^+(\sigma,t) = v_s(s_{*1} - 0, t), \qquad \lim_{\sigma\to 0} v_{w1\sigma}^-(\sigma,t) = v_s(s_{*1} + 0, t),\\ &\lim_{\sigma\to 0} v_{w2\sigma}^+(\sigma,t) = -v_s(s_{*2} - 0, t), \qquad \lim_{\sigma\to 0} v_{w2\sigma}^-(\sigma,t) = -v_s(s_{*2} + 0, t). \end{aligned} \tag{10}$$

Taking into account (7)–(10), the intensities of the vortices $\gamma_{w1}(0,t)$ and $\gamma_{w2}(0,t)$ trailing from the airfoil L and the vortex wakes L_{w1} and L_{w2} are defined by the formulas

$$\gamma_{w1}(0,t) = v_s(s_{*1} + 0, t) - v_s(s_{*1} - 0, t) = \gamma(s_{*1} - 0, t) + \dot{s}_{*1}(t),$$

$$\gamma_{w2}(0,t) = -v_s(s_{*2} + 0, t) + v_s(s_{*2} - 0, t) = \gamma(s_{*2} + 0, t) + \dot{s}_{*2}(t).$$

The velocities of the vortices $w_1(t)$ and $w_2(t)$ trailing to the wakes L_{w1} and L_{w2} are determined by the relative velocity of vortices at $\sigma \to 0$:

$$
\begin{aligned}
w_1(t) &= [v_s(s_{*1} - 0, t) + v_s(s_{*1} + 0)]/2 - v_{es}(s_{*1}, t) = -\gamma_{w1}(0, t)/2, \\
w_2(t) &= -[v_s(s_{*1} - 0, t) + v_s(s_{*1} + 0)]/2 + v_{es}(s_{*1}, t) = \gamma_{w2}(0, t)/2.
\end{aligned}
\tag{11}
$$

5. We now consider condition (1). Differentiating expression (5) with respect to s, we have

$$
\frac{\partial p}{\partial s} = -\rho\Big\{\frac{\delta v_s}{\delta t} + (v_s - v_{es})\,\frac{\partial}{\partial s}\,(v_s - v_{es}) - \frac{\partial}{\partial s}\Big(\frac{v_e^2}{2}\Big)\Big\}. \tag{12}
$$

Here $v_e = |\boldsymbol{v}_e|$, where $\boldsymbol{v}_e$ is the velocity of motion of the flow separation point. By the definition, the tangential v_{es} and normal v_{en} components of the velocity $\boldsymbol{v}_e$ are equal to

$$
v_{es} = U_s + \dot{s}_{*k}, \qquad v_{en} = U_n. \tag{13}
$$

In view of (7) and (13), at the point $s = s_{*1} - 0$, we have

$$
\frac{\delta v_s}{\delta t} = \dot{U}_s(s_{*1}, t) - \dot{\gamma}(s_{*1} - 0, t),
$$

$$
(v_s - v_{es})\,\frac{\partial}{\partial s}\,(v_s - v_{es}) = [\gamma(s_{*1} - 0, t) + \dot{s}_{*1}(t)]\,\frac{\partial}{\partial s}\,\gamma(s, t)\Big|_{s=s_{*1}-0},
$$

$$
\frac{\partial}{\partial s}\Big(\frac{v_e^2}{2}\Big) = [U_s(s_{*1}, t) + \dot{s}_{*1}(t)]\,\frac{\partial}{\partial s}U_s(s, t)\Big|_{s=s_{*1}} + U_n(s_{*1}, t)\,\frac{\partial}{\partial s}U_n(s, t)\Big|_{s=s_{*1}}.
$$

Similarly, at the other flow separation point at $s = s_{*2} + 0$, we have

$$
\frac{\delta v_s}{\delta t} = \dot{U}_s(s_{*2}, t) - \dot{\gamma}(s_{*2} + 0, t),
$$

$$
(v_s - v_{es})\,\frac{\partial}{\partial s}\,(v_s - v_{es}) = [\gamma(s_{*2} + 0, t) + \dot{s}_{*2}(t)]\,\frac{\partial}{\partial s}\,\gamma(s, t)\Big|_{s=s_{*2}+0},
$$

$$
\frac{\partial}{\partial s}\Big(\frac{v_e^2}{2}\Big) = [U_s(s_{*2}, t) + \dot{s}_{*2}(t)]\,\frac{\partial}{\partial s}\,U_s(s, t)\Big|_{s=s_{*2}} + U_n(s_{*2}, t)\,\frac{\partial}{\partial s}\,U_n(s, t)\Big|_{s=s_{*2}}.
$$

Substituting these expressions into (12) and requiring the satisfaction of condition (1), we arrive at the following equations for the arc coordinates $s_{*1}(t)$ and $s_{*2}(t)$ of the flow separation points from the smooth contour:

$$
\begin{aligned}
&\dot{U}_s(s_{*1}, t) - \dot{\gamma}(s_{*1} - 0, t) + [\gamma(s_{*1} - 0, t) + \dot{s}_{*1}(t)]\,\frac{\partial}{\partial s}\,\gamma(s, t)\Big|_{s=s_{*1}-0} \\
&- [U_s(s_{*1}, t) + \dot{s}_{*1}(t)]\,\frac{\partial}{\partial s}\,U_s(s, t)\Big|_{s=s_{*1}} - U_n(s_{*1}, t)\,\frac{\partial}{\partial s}\,U_n(s, t)\Big|_{s=s_{*1}} = 0;
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&\dot{U}_s(s_{*2}, t) - \dot{\gamma}(s_{*2} + 0, t) + [\gamma(s_{*2} + 0, t) + \dot{s}_{*2}(t)]\,\frac{\partial}{\partial s}\,\gamma(s, t)\Big|_{s=s_{*2}+0} \\
&-[U_s(s_{*2}, t) + \dot{s}_{*2}(t)]\,\frac{\partial}{\partial s}\,U_s(s, t)\Big|_{s=s_{*2}} - U_n(s_{*2}, t)\,\frac{\partial}{\partial s}\,U_n(s, t)\Big|_{s=s_{*2}} = 0.
\end{aligned}
\tag{15}
$$

Equations (14) and (15) are nonlinear differential equations of the first order for $s_{*1}(t)$ and $s_{*2}(t)$. The initial conditions are determined by specifying the arc coordinates $s_{*1}(0)$ and $s_{*2}(0)$ of the points of the contour L at which the hydrodynamic pressure reaches the local minimal value in the steady-state flow and $\dot{s}_{*1}(0) = \dot{s}_{*2}(0) = 0$. The functions $U_s(s, t)$ and $U_n(s, t)$ are specified and the intensity of the vortex sheet $\gamma(s, t)$ is determined by solving the corresponding nonlinear initial-boundary-value problem which, in turn, depends on the positions $s_{*1}(t)$ and $s_{*2}(t)$ and the velocities of motion $\dot{s}_{*1}(t)$ and $\dot{s}_{*2}(t)$ of the flow separation points. Thus, Eqs. (14) and (15) represent only part of the nonlinear equations that are to be solved jointly for the particular initial-boundary-value problem of the unsteady separated flows past a smooth airfoil.

We note that at the points $s_{*1}+0$ and $s_{*2}-0$ on the other edge of the cuts L_{w1} and, L_{w2},

$$\frac{\partial p}{\partial s} = -\rho\Big\{\dot{U}_s(s_{*k},t) + \ddot{s}_{*k}(t) - \frac{\partial}{\partial s}\Big(\frac{v_e^2}{2}\Big)\Big|_{s=s_{*k}}\Big\}, \qquad k=1,2. \tag{16}$$

An analysis of expression (16) leads to the following two remarks. First, it cannot be regarded as the equation for the flow separation point since the coordinate $s_{*k}(t)$ is related only to the law of motion of the contour. Second, in the general case, the right side of expression (16) is different from zero. Thus, $\partial p/\partial s$ undergoes a discontinuity in passing through the vortex wake L_w at the flow separation point, and the assumption of the continuity of $\partial p/\partial s$ made in [10] is invalid.

6. Let us elucidate some details concerning the separation of the vortex wakes L_{w1} and L_{w2} from the contour L. According to Kelvin's theorem on the constancy of velocity circulation around a closed contour encompassing the contour L and the vortex wakes,

$$\frac{d}{dt}\,\{\Gamma(t) + \Gamma_{w1}(t) + \Gamma_{w2}(t)\} = 0,$$

$$\Gamma(t) = -\int\limits_L v_s(s,t)\,ds = \int\limits_L \gamma(s,t)\,ds, \qquad \Gamma_{wk}(t) = \int\limits_{L_{wk}} \gamma_{wk}(\sigma,t)\,d\sigma, \quad k=1,2. \tag{17}$$

Here $\Gamma(t)$ and $\Gamma_{wk}(t)$ is the counter-clockwise velocity circulation around the contours L and L_{wk}. We shall show that

$$\frac{d\Gamma_{w1}}{dt} = -\frac{1}{2}\,\gamma_{w1}^2(0,t), \qquad \frac{d\Gamma_{w2}}{dt} = \frac{1}{2}\,\gamma_{w2}^2(0,t). \tag{18}$$

The time derivative of the velocity circulation around L_{wk} is linked to the discontinuity of the velocity potential at the point of separation of the vortex wake by the relation

$$\frac{d}{dt}\,\Gamma_{wk}(t) = \frac{\delta}{\delta t}\,[\varphi(s_{*k}-0,t) - \varphi(s_{*k}+0,t)]. \tag{19}$$

In turn, the derivative of the velocity potential discontinuity with respect to t can be found from the condition of pressure continuity in passing through L_{wk}. From (5) it follows that this condition can be written as

$$\frac{\delta}{\delta t}\,\varphi(s_{*k}-0,t) + \frac{1}{2}\,[v_s(s_{*k}-0,t) - v_{es}(s_{*2},t)]^2$$
$$= \frac{\delta}{\delta t}\,\varphi(s_{*k}+0,t) + \frac{1}{2}\,[v_s(s_{*k}+0,t) - v_{es}(s_{*k},t)]^2. \tag{20}$$

In the vicinity of the point of separation of the vortex wakes L_{w1} and L_{w2}, we have

$$v_s(s_{*1}+0,t) - v_{es}(s_{*1},t) = 0, \qquad v_s(s_{*2}-0,t) - v_{es}(s_{*2},t) = 0,$$

$$[v_s(s_{*1}-0,t) - v_{es}(s_{*1},t)]^2 = \gamma_{w1}^2(0,t), \qquad [v_s(s_{*2}+0,t) - v_{es}(s_{*2},t)]^2 = \gamma_{w2}^2(0,t).$$

Substituting these expressions into (20), we obtain

$$\begin{aligned}\frac{\delta}{\delta t}\,[\varphi(s_{*1}-0,t) - \varphi(s_{*1}+0,t)] &= -\frac{1}{2}\,\gamma_{w1}^2(0,t),\\ \frac{\delta}{\delta t}\,[\varphi(s_{*2}-0,t) - \varphi(s_{*2}+0,t)] &= \frac{1}{2}\,\gamma_{w2}^2(0,t).\end{aligned} \tag{21}$$

From (19) and (21) it follows that the time derivatives of the velocity circulation around the vortex wakes L_{w1} and L_{w2} indeed satisfy relations (18). Thus, for the separation of the two vortex wakes from the smooth airfoil, the derivatives $d\Gamma_{w1}/dt$ and $d\Gamma_{w2}/dt$ have different signs, and condition (17) is satisfied for an arbitrary time dependence of the velocity circulation around the contour L. The separation of the vortex wake L_{wk} is possible only for a positive value of the separation velocity w_k. From (11) it follows that for the separation of the two wakes, it is necessary that the conditions $\gamma_{w1}(0,t) < 0$ and $\gamma_{w2}(0,t) > 0$ be satisfied.

Let us now find the features of the model of separated flow with one vortex wake trailing from the smooth airfoil L. In the general case of motion of the airfoil, the velocity circulation cannot be a monotonic function of time. Therefore, the model for separated flow past a smooth airfoil with one trailing vortex wake should provide the possibility of replacing the vortex wake L_{w1} by L_{w2} and vice versa. The replacement of one vortex wake by the other occurs at the times when the time derivative of the velocity circulation vanishes $[d\Gamma(t)/dt = 0]$. In the time interval in which $d\Gamma(t)/dt > 0$, a vortex wake of type L_{w1} trails from the airfoil L and at $d\Gamma(t)/dt < 0$, a vortex wake of type L_{w2} occurs. This separated flow model was used, in particular, in [11].

It should be noted that a similar situation arises for unsteady flow of an airfoil with an angular rear lip. In this case, the vortex wake separates from the profile along the tangent to alternatively one or the other face of the angular lip, depending on the sign of the derivative $d\Gamma/dt$ [12].

In summary, we note the main findings of the work.

1. The necessary criteria for the separation of unsteady ideal fluid flow from a closed smooth airfoil were formulated.

2. A simple and effective formula for the calculation of the hydrodynamic pressure at the points of the moving airfoil was derived.

3. A nonlinear differential equation of the first order was obtained for the arc coordinate of the flow separation point, which is a function of time and depends on the velocity field around the contour and the law of its motion is obtained.

4. An analysis was performed of the main features of the model for the unsteady separated flow past a smooth airfoil with one and two vortex wakes.

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